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INSTABILITIES OF HIGHER DIMENSIONAL COMPACTIFICATIONS

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ABSTRACT

We consider various schemes for cosmological compactification of higher dimensional theories. We discuss possible instabilities which drive the ground state with static internal space to de Sitter-like expansion of all dimensions. These instabilities are due to semiclassical barrier penetration and classical thermal fluctuations. For the case of the ten dimensional Chapline-Manton action, it is possible to avoid such difficulties by balancing one-loop Casimir corrections against monopole contributions from the field strength H_{MNP} and fermionic condensates.

1. Introduction

Attempts to unify gravity with the strong and electro-weak interactions have lead to a great deal of interest in theories with extra spatial dimensions[1]. The most promising theories of this type are superstring theories which appear to be consistent only in ten dimensions. However, any higher dimensional theory must incorporate the fact that at energies presently accessible to accelerators, which can probe distances of order 10^{-16}cm, extra spatial dimensions are unobservable. In addition, these extra dimensions must be static since if they vary, fundamental constants will vary. For example, variation in the fine structure constant can affect the amount of primordial helium produced at the time of nucleosynthesis[2]. Requiring that these abundances lie within acceptable limits constrains

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in either superstring or Kaluza-Klein theories the size of the extra dimensions at nucleosynthesis to be very near its equilibrium value. Since the only scale in these theories is the Planck scale, it is not unreasonable to suppose that the universe has been effectively four dimensional since $\sim 10^{-42}$ seconds after the big bang. At present, it is not known how the universe evolved from say ten dimensions to four dimensions plus some presumably compact internal space. It appears that such an evolution would require a change in the topology of space-time, a highly non-perturbative effect. However, once the universe has this product space structure it is possible to study in a cosmological setting how it evolves so that the extra dimensions presently form a small, static internal space.

Our approach to studying the cosmological evolution of the extra dimensions is to begin with matter fields defined on a $4+D$ dimensional manifold with the *ansatz* that this manifold has the product space structure $M^{4+D} = R^1 \times Q^3 \times \prod_{i=1}^{\alpha} S_i^d$, where Q^3 is the physical 3-space with radius a, $D = \alpha d$ and the internal radii are $b_1, \ldots, b_\alpha$. If we introduce a $4+D$ dimensional cosmological constant Λ^{4+D}, and for the present take $\alpha = 1$, then by balancing Λ^{4+D} against the vacuum stress energy of the matter fields (gravity will be treated classically here) the internal D-sphere is stable against small perturbations around some equilibrium value, b_0, of the internal radius. This configuration comes about by requiring that the minimum energy state be static and have a vanishing Λ^4.

Compactification stabilized due to the vacuum stress energy of quantum fluctuations due to non-trivial boundary conditions, is analagous to the Casimir effect in quantum electrodynamics[3)], while compactification due to classical stress energy can arise from the existence of monopole configurations for gauge and matter fields[4)].

Though these stabilization schemes are perturbatively stable, it has been demonstrated that the ground state manifold is semiclassically unstable–there is a nonzero probability for decay via quantum tunneling through a potential barrier[5)]. In addition, at non-zero temperature there exists the possibility of classically rolling over the barrier due to thermal fluctuations[6)]. In both cases, the instability is characterized by a de Sitter-like expansion of all dimensions.

The semiclassical instability is the result of adding a cosmological constant to the action. However for higher dimensional supergravity theories, such the field theoretic limit of the heterotic string, one cannot have a cosmological constant since this explicitly breaks supersymmetry. One can achieve a stable compactification in such theories by balancing Casimir-like one-loop quantum effects against monopole configurations which include contributions from fermionic condensates[7)].

In section 2 we will discuss stabilization of the internal space using the higher dimensional cosmological constant and Casimir contribution (monopole contributions give similar results). In section 3 we will discuss the situation for ten dimensional supergravity where stabilization of the internal space is brought about by balancing Casimir and monopole contributions.

2. Semi-classical and Thermal Instabilities

The free energy for non-interacting spinless matter fields in thermal equilibrium at temperature T is

$$\beta T = \frac{1}{2}\ln\,\det(-\Box_{4+D} + \mu^2). \tag{1}$$

Here the product space manifold, $S^1 \times S^3 \times S^D$, is Euclidean with the time direction compactified to a circle of radius $\beta/2\pi$. After regularization, and generalizing to a set of spinless, noninteracting fields, the free energy can be approximated[8)] in the "flat-space" limit, $a >> b$, as

$$F = \frac{\Omega_3}{b^4}\left[c_1 - c_2(2\pi bT)^4 - c_3(2\pi bT)^{4+D}\right]. \tag{2}$$

Here Ω_3 is the volume of the physical 3-sphere, c_1 is the Casimir coefficient c_N of Candelas and Weinberg, while c_2 and c_3 are thermal terms. Equation (2.2) has the correct high ($T > 1/2\pi b$) and low ($T < 1/2\pi b$) temperature limits for the free energy. For our product space metric, the stress-energy tensor has the form $T_{MN} = \mathrm{diag}(\rho,\ p_3\tilde{g}_{ij},\ p_D\tilde{g}_{mn})$ and the components of T_{MN} can be obtained from Eq. (1) using standard thermodynamic relations generalized to higher dimensions. Plugging these results into Einstein's equations, one finds that the equation of motion for the b scale factor can be written

$$\frac{\ddot{b}}{b} + (D-1)\frac{\dot{b}^2}{b^2} + 3\frac{\dot{b}\dot{a}}{ba} = -\frac{(D-1)}{b^2} + \frac{D(D-1)}{(D+4)} \times \left[\left[b_0^{-2} + \frac{4}{D}\frac{b_0^{D+2}}{b^{D+4}}\right] + \frac{b_0^{D+2}}{b^{D+4}}\frac{c_3}{c_1}(2\pi bT)^{(D+4)}\right]. \tag{3}$$

This can be recast as an equation of motion for a scalar field minimally coupled to gravity in four dimensions, with potential

$$V(\Phi,T) = \frac{(D-1)\Lambda m_{Pl}^2}{8\pi(D+2)}\left[\frac{(D+4)}{(D-2)}(\Phi^{(2/D)(D-2)} - 1) + \Phi^{-8/D} - \left[1 + \frac{c_3}{c_1}(2\pi b_0 T)^{D+4}\right]\Phi^2 + \frac{c_3}{c_1}(2\pi b_0 T)^{D+4}\right]. \tag{4}$$

At temperatures less than some critical temperature, V is unbounded from below for large values of Φ and has a barrier which separates this region from the vacuum ($\Phi = \Phi_0$) with static internal space of radius $b = b_0$ and zero cosmological constant. The lifetime of the compactified state can be estimated in a straightforward fashion. The semi-classical decay rate per unit 4-volume is $\Gamma = m^4\exp(S_4)$ where S_4 is the four dimensional euclidean action for the field Φ and m is a determinant with mass scale m_{Pl}. For $T = 0$ and $D = 7$, we can approximate V with $V(\bar{\Phi}) \approx 0.093\Lambda\bar{\Phi}^2 - 0.159\Lambda\bar{\Phi}^3/m_{Pl}$ (here Λ is Λ^{4+D}) and the tunnel action is $S_4 \approx 165 m_{Pl}^2/\Lambda$. The decay amplitude becomes of order one when $\tau \approx m_{Pl}^{-1}\exp(41 m_{Pl}^2/\Lambda)$ so that the compactification lifetime will be longer than the present age of the universe, $\tau > H_0^{-1}$, for values of $\Lambda \leq 0.3 m_{Pl}^2$ which corresponds to values of $b_0 \geq 11 l_{Pl}$.

At finite temperature, $V(\Phi,T)$ has a local minimum Φ_0 when $T < T_{crit}$ while for $T > T_{crit}$, $V(\Phi,T)$ monotonically decreases. The barrier height drops as T increases, vanishing at $T = T_{crit}$. As might be expected, classical thermal fluctuations of the compactified space are important for temperatures $T > 1/2\pi b$. If we require that $\Lambda^{4+D} \leq 0.3 m_{Pl}^2$, then $T_H = 1/2\pi b \leq 1.49 \times 10^{-2} m_{Pl}$, while $T_{crit} \leq 2 \times 10^{-2} m_{Pl}$. This narrow region of interesting temperatures should not be surprising since $V(\Phi, T)$ is such a strong function of T. The finite temperature vacuum decay rate is $\Gamma \approx \beta^{-4}\exp[-\beta S_3(\Phi, T)]$. Now the relevent scale for the determinant is $1/\beta$ and the fact that at finite temperature euclidean time is periodic in β allows us to write $S_4 = \beta S_3$ and S_3 is the three dimensional euclidean action (free energy).

We see that if $\Phi = \Phi_0$ when $T > T_{crit}$, stabilization of the extra dimensions is impossible. In the region $T_{crit} \geq T_{comp} \geq T_H$, where T_{comp} is the temperature at which $\Phi = \Phi_0$, then the decay rate is large only for $T_{comp} \sim T_{crit}$ and to avoid a destabilizing thermal fluctuation, the initial entropy must be made small. This corresponds to a small value for Λ^{4+D} which in turn implies a larger radius for the internal space. Though the decay rate is large only for $T_{comp} \sim T_{crit}$, we should note that $T_{crit} << m_{Pl}$ so that if compactification takes place near the Planck scale, it seems difficult to have hot initial conditions in these models.

3. Stability for Ten-dimensional Supergravity

Though the instabilities discussed in the last section can be avoided, or at least postponed by adjusting parameters, it would be preferable if such fine tuning were not necessary. That $V(\Phi, T)$ is unbounded from below for large values of Φ is a consequence of including a cosmological constant in the higher dimensional action. Since we wish to consider compactifications in more realistic supersymmetric models, alternate compactification schemes which do not include Λ^{4+D} and so may not contain instabilities should be investigated.

Type I or heterotic string theories contain $N = 1$ supersymmetry coupled to $N = 1$ super-Yang-Mills in ten dimensions. The action contains an antisymmetric rank-2 tensor with an accompanying three-index field strength H. Returning to our product space metric with 2 internal 3-spheres, we can use the Freund-Rubin *ansatz*[9] for the field strength H_{MNP}, giving it a monopole configuration on each of the $i = 1,\ 2$ internal 3-spheres:

$$H_{MNO} = \sqrt{g^{(3)}}\epsilon_{m_i n_i p_i} f^{(i)}(t), \tag{5}$$

and setting it to zero on the external space. The Bianchi identities then tell us that $f^{(i)}(t) = f_0^{(i)}/b_i^d(t)$. The vacuum stress energy due to monopole configurations will scale as $1/b_i^{2d}$.

For a manifold $R \times S^3 \times S^D$, with $a \to \infty$, the Casimir contribution to the vacuum stress energy has the form

$$F = \Omega_3 \left[\frac{A + A'\ln(2\pi\rho^2)}{b^4}\right] \tag{6}$$

Here, A and A' are calculable coefficients, $\rho^2 = \mu^2 b^2$, and μ is a regularization scale. In odd dimensions, A' vanishes so that F does not explicitly depend on an undetermined parameter. For our purposes, we can neglect the logarithmic dependence on the radius and set the numerator equal to a constant.* Using our product space *ansatz*, we write the Casimir free energy as

$$F = \Omega_3 \sum_{i=1}^{2} \frac{A^{(i)}}{b_i^4}. \tag{7}$$

Since the monopole and Casimir energies scale differently, there will be non-trivial values of the b_i for which the internal spaces are static. However, for models which do not contain fermionic condensates, Minkowski space appears as a perturbatively unstable point for the equations of motion.

In the case of the ten dimensional Chapline-Manton action[10)], it is possible to obtain a stable compactification. Consider the bosonic part of this action including gluino and subgravitino couplings. We set the Yang-Mills field strength to zero and the dilaton to a constant $\sigma = \sigma_0$. The internal space is a product of two 3-spheres. In addition we impose the Freund-Rubin condition for H_{MNP} and fermionic condensates. These are related to each other in a non-trivial fashion through the dilation field equation

$$e^{-\sigma_0}(H_{MNP})^2 = \frac{3}{2} e^{-\sigma_0/2} H_{MNP}(Tr\bar{\chi}\Gamma^{MNP}\chi) \tag{8}$$

After adding Casimir terms, setting $b_1 = b_2 = b$, and rescaling coefficients we find that the b equation of motion can be written

$$\frac{\ddot{b}}{b} + 5\frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}\dot{b}}{ab} = -\frac{2}{b^2} + \frac{4A}{3b^{10}} + \frac{c'}{b^6} \tag{9}$$

The coefficient c' is a function of the monopole strengths of H and the fermionic condensates. In terms of an effective four dimensional scalar field $\phi = ln(b/b_0)$, we can define an equation of motion with potential

$$V(\phi) = b_0^{-2}\left[-e^{-2\phi} + \frac{c'}{6b_0^4}e^{-6\phi} + \frac{(2b_0^4 - c')}{10b_0^4}e^{-10\phi} + \frac{12b_0^4 - c'}{15b_0^4}\right]. \tag{10}$$

The critical points for this potential are $\phi_1 = 0$ and $\phi_2 = \frac{1}{4}\ln[-2b_0^4/(2b_0^4 - c')]$ with $2b_0^4 < c'$, $c' > 0$. For ϕ_1 there exists a minimum at b_0 when $4b_0^4 > c'$, $c' > 0$ or for $c' < 0$. For ϕ_2 we find that Minkowski space is once again a maximum. To realize $\phi_1 = 0$, set the gluino monopole strenth equal to the negative of the H monopole strength. Then $c' = 6b_0^4/5$ and the effective four dimensional cosmological constant vanishes. No fine tuning is needed to realize a stable compactification but in this approach, stability away from the $b_1 = b_2$ line in phase space is unknown.

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